PAPER • **OPEN ACCESS**

Impact of the Method of Analysing Post-Tensioned Flat Slabs on the Amount of Prestressing

To cite this article: Rafa Szydowski and Barbara abuzek 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **473** 012046

IOP Conf. Series: Materials Science and Engineering 473 (2019) 012046 doi:10.1088/1757-899X/473/1/012046

Impact of the Method of Analysing Post-Tensioned Flat Slabs on the Amount of Prestressing

Rafał Szydłowski[1], Barbara Łabuzek[1]

[1]Cracow University of Technology, Warszawska 24 Street, Cracow 31-155

Abstract. The post-tensioned slabs have been using for many years. Over the years, many guidelines regarding design methods and calculations have been issued. There are several methods of static analysis and obtained results interpretation. It should be noted that the selection of method has a large impact on the final results in terms of required amount of presstresing steel. It is a great importance for economic aspects of realization of the post-tensioned slabs. In this paper the results of analysis of flat post-tensioned slab are presented. The calculations were carried out using two methods: the equivalent frame method and final elements method. The differences between these two methods and the impact of the selected method on the final usage of the prestressing steel are described in detail.

1. Introduction

The flat slab, i.e. slab without any beams, it is the most commonly used form of slab in concrete structures. A higher slab thickness and higher consumption of reinforcing steel compared to traditional slab-beam systems are compensated by advantages such as lower total height of floor, free space under the slab (lack of beams), no obstacles in running the installation and much easier and faster execution of formwork and reinforcement which translates into lower cost of realization.

The flat post-tensioned slabs are particularly attractive in terms of architecture. Application of prestressed concrete technology allows for enhancement the spacing of columns to 12 meters and in some exceptional cases even to 14-16 meters. The attempts to improve them have been made for a few years at the Cracow University of Technology [1-3].

2. The methods of slabs analysis

Analysis of flat slabs presents many difficulties. It is related to peak value of bending moments over the columns as a results of point supports. This type of structure can be analyzed with Finite Elements Methods or with one of simplified method, as equivalent frame method or grillage method. The value of bending moments near the columns is several times larger than in the center point between the columns. Thus, interpretation of results near the columns is difficult. Further obstacles are encountered in the case of post-tensioned flat slabs. Active reinforcement generates additional loads that have to be added to the numerical model which is time-consuming process. Consequently, the selection of method of analyse is more important than in case of reinforced slabs. The different methods require different work time arrangements, giving different amounts of prestressing steel which assure meeting the conditions of limit states. The two most popular methods and the results obtained with their use are presented in the further part of the work.

2.1. Equivalent frames method

In the equivalent frame method the slab is represented as a set of two-dimensional frames. Each of the frames consists of the columns located above and below the slab as well as horizontal beam representing the strip of slab between the lines of zero shear (figure 1). The number and arrangement of frames should be taken so that the whole slab is covered with frames. In case of using the equivalent frame method, for estimation of concrete cross-section stresses the bending moments are averaged at the total width of the frame. Therefore, this method does not take into account elastic moment distribution in the transverse direction.

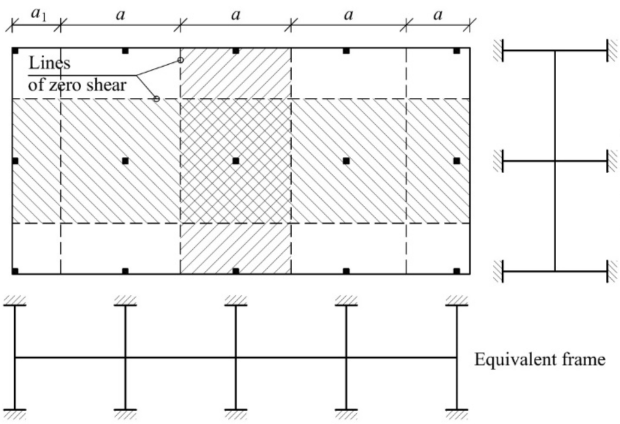

Figure 1. Elastic load distribution effects and slab division on equivalent frames.

2.2. *Finite elements method*

This method allows to analyze the slab as a whole three-dimensional structure. The three-dimensional analysis is possible through the precise modeling with taking into consideration elastic load distribution and actual distribution of prestressing tendons. However, if the calculation are not carrying out with the use of advanced computational program this method is a more time-consuming method than it requires.

2.3. *Limit stresses*

As a result of other distributions of bending moments obtained from both methods the limit stress values are slightly different. Recommendations of *Concrete Society* [4] gives the moments averaging ways for both methods as well as the limit concrete cross-section stress values. For equivalent frame method the bending moment in whole slab width between zero shear lines is considered in all cross-sections. In case of FEM the paper [4] determinates the design strips. These are defined as the entire width of the strips between the lines of zero shear in the span cross-section (*a* in figure 1) and 0.4 of strips width in the column cross-section. Bending moments obtained from FEM analysis are averaged on the design strips. The values of limit stresses for both methods are set in table 1. Presented values are matched to the moments averaging process in each of the methods. Limit stresses for using design strips (FEM) are significantly higher than for full panel width. It results from the other method of averaging moments.

Table 1. Limit stresses in flat slab according to [4].

<table>
<tr><th rowspan="2">Location</th><th colspan="2">For full panel width</th><th colspan="2">For using “design strips”</th></tr>
<tr><th>In compression</th><th>In tension</th><th>In compression</th><th>In tension</th></tr>
<tr><td>Support</td><td>$0.3f_{ck}$</td><td rowspan="2">$0.9f_{ctm}$</td><td rowspan="2">$0.4\ f_{ck}$</td><td rowspan="2">$1.2f_{ctm}$</td></tr>
<tr><td>Span</td><td>$0.4f_{ck}$</td></tr>
</table>

3. Slab geometry and characteristic

Analysed slab is presented in figure 2. The columns with cross-section of 0,4×0,4 m are arranged on regular gird 8,5×8,5m. The slab thickness is 250mm. C40/45 class concrete was used. Figure 2 presents the slab geometry as well as its division into equivalent frames with their designation.
The following external loads were adopted in slab calculation: self-weight (25kN/m^3), floor layers (1,80kN/m^2) and service load (3,9kN/m^2).

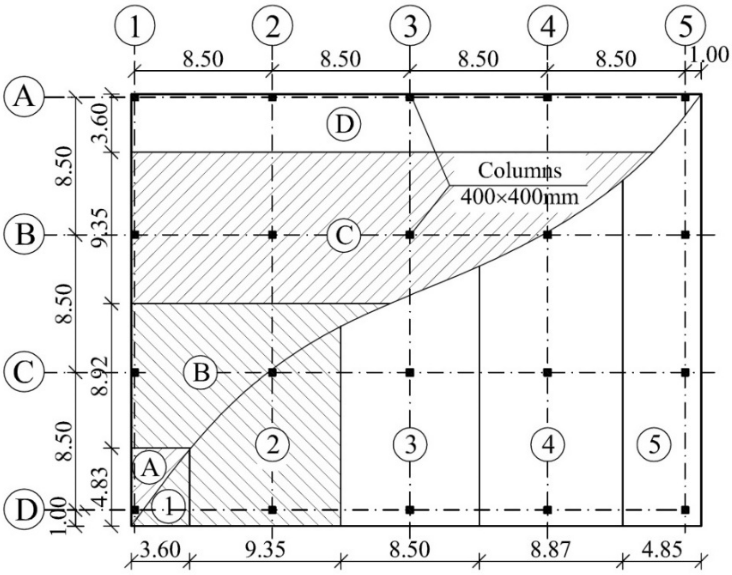

Figure 2. Slab geometry and its division into equivalent frames.

4. The results of analysis

Prestressing with unbonded 15.2mm tendons was assumed. The initial force in tendon is 220kN. Tendon profiles in both directions are presented in figure 3. Effective tendon force $P_{eff} = 0.8P_0 = 176$kN was assumed in analysis. The number of tendons in equivalent frames and tendon arrangement in the slab were adopted so as to meet the limit stresses. The number of tendons in equivalent frames is presented in table 2 and tendon arrangement in the slab (analyzed with FEM) with equivalent load from prestressing in Y direction is shown in figure 4.

Table 3 presents the concrete stresses in two equivalent frames (equivalent frame method) and in two axis for FEM, one in each direction. The axis B and 2 were considered for both of methods. The stress values obtained from the methods are presented in table 3, results of FEM analysis were placed into brackets. Besides the calculated results the table also includes the limit stress values. The limit stress value was used to set the quantity of tendons, which presented in table 2.

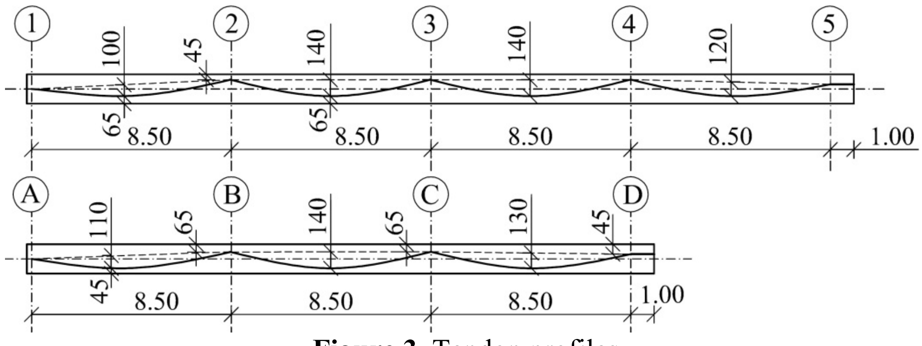

Figure 3. Tendon profiles.

Table 2. Number of prestressing strands in equivalent frames.

Frame	1	2	3	4	5	A	B	C	D
Strand No.	8	22	19	21	16	13	33	32	18

Table 3. Concrete cross-section stresses for equivalent frames method and FEM (in brackets).

Layer	Frame B Location	Dead and service load	Prestress -ing	Final	Frame 2 Location	Dead and service load	Prestress -sing	Finall	Ultimate value
Top	1-2	4.28 (4.95)	0.67 (1.03)	4.95 (5.98)	A-B	3.82 (4.85)	0.22 (0.07)	4.04 (4.92)	14.0 (14.0)
Bottom		-4.28 (-4.95)	3.27 (4.92)	-1.01 (-0.03)		-3.82 (-4.85)	2.79 (4.34)	-1.03 (-0.50)	-2.88 (-3.84)
Top	2	-7.46 (-11.57)	4.60 (9.32)	-2.87 (-2.25)	B	-6.30 (-11.51)	3.54 (8.02)	-2.76 (-3.49)	-2.88 (-3.84)
Bottom		7.46 (11.57)	-0.66 (-3.80)	6.81 (7.77)		6.30 (11.51)	-0.53 (-3.91)	5.77 (7.60)	10.5 (14.0)
Top	2-3	3.06 (3.09)	-0.01 (-0.06)	3.05 (3.03)	B-C	2.36 (2.73)	0.15 (0.34)	2.50 (3.06)	14.0 (14.0)
Bottom		-3.06 (-3.09)	3,95 (5.85)	0,89 (2.76)		-2.36 (-2.73)	2.86 (3.92)	0.50 (1.19)	-2.88 (-3.84)
Top	3	-6,06 (-9.69)	5,24 (11.10)	-0,83 (1.41)	C	-6.22 (-11.27)	3.34 (7.46)	-2.88 (-3.81)	-2.88 (-3.84)
Bottom		6.06 (9.69)	-1,30 (-5.73)	4,76 (3.96)		6.22 (11.27)	-0.33 (-3.28)	5.89 (7.99)	10.5 (14.0)
Top	3-4	3.07 (3.16)	-2.04 (-0,14)	1.03 (3.02)	C-D	3.73 (4.65)	0.43 (0.48)	4.16 (5.13)	14.0 (14.0)
Bottom		-3.07 (-3.16)	4.01 (5.96)	0.94 (2.80)		-3.73 (-4.65)	2.58 (3.98)	-1.15 (-0.68)	-2.88 (-3.84)
Top	4	-7.01 (-11.37)	4.24 (8.88)	-2.77 (-2.49)	D	-2.73 (-3.19)	2.79 (7.87)	0.06 (4.68)	10.5 (14.0)
Bottom		7.01 (11.37)	-0.30 (-3.29)	6.71 (8.08)		2.73 (3.19)	0.22 (0.25)	2.95 (3.43)	-2.88 (-3.84)
Top	4-5	4.18 (4.75)	1,02 (1.63)	5.20 (6.38)					14.0 (14.0)
Bottom		-4.18 (-4.75)	2,92 (4.39)	-1.26 (-0.36)					-2.88 (-3.84)
Top	5	-3.19 (-2.87)	3.65 (9.87)	0.46 (3.49)					-2,88 (-3.84)
Bottom		3.19 (2.87)	0.29 (0.61)	3.47 (7.00)					10.5 (14.0)

The stress values for decisive cross-sections are marked in red. It can be observed, that the cross-section at axis C was crucial in both of methods. Tensile stresses are decisive. It is interesting, that the amount of tendons applied in frame 2 is 22 and in frame B is 33 for equivalent frames method. While the number of tendons for FEM method is 30 and 40 respectively. It means, that the amount of prestressing steel needed for meeting the acceptable concrete stress conditions obtained from FEM analysis is 36% higher than calculated from equivalent frames method.

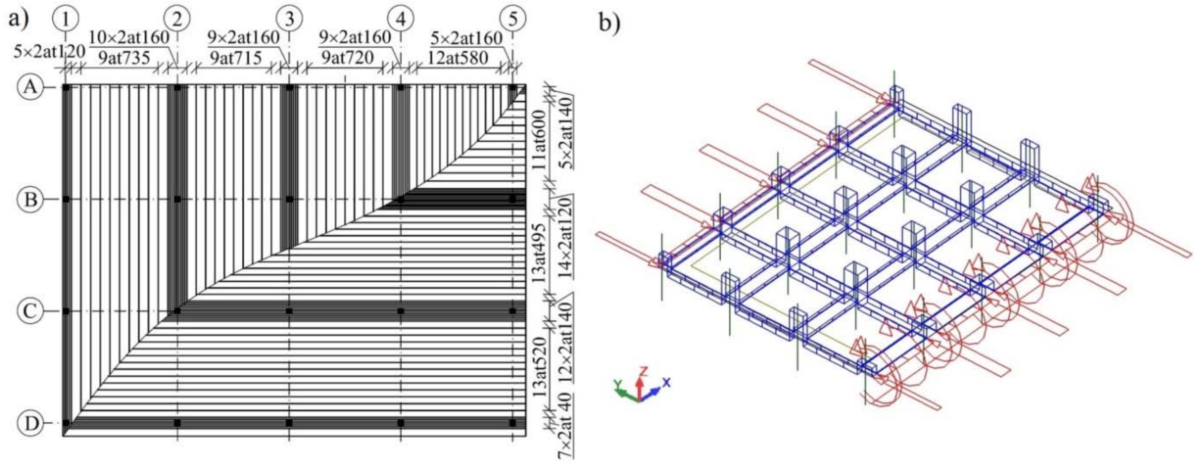

Figure 4. Tendon arrangement (a), equivalent load for tendons in Y direction (b).

5. Conclusions

This paper presents the results of post-tensioned slab analysis based on two methods: FEM and equivalent frames method. FEM method treats the structure more accurately considering the elastic distribution of moments in the transverse direction. It is more time-consuming compared to simplified equivalent frames methods. In spite of these, the simpler method turned out to be more economical in the amount of prestressing needed. It is due to the way of averaging the peak moment above the columns, delivered in recommendations [4]. The range of integration and averaging moments in FEM is larger than one in the equivalent frame method. Thus, unit moments and cross-section stresses obtain in FEM are higher. It is not recompensed by higher values of limit stresses.

The quantitative difference in the values obtained cannot be generalized, it is strongly depended on the prestress arrangement and the way of its modelling. It is indisputable, the simpler and less-time consuming method allows to design more economically. It should be noted, that recommendations used are valid for elastic analysis of the structures. Non-liner analysis taking into consideration crack appearance and bending moments redistribution will give other effects. It will allow to take advantage of FEM analysis and to design more economically.

References

[1] Derkowski W, Skalski P 2017 *Procedia Eng.* **193** 176-83
[2] Szydlowski R, Labuzek B 2017 *Post-Tensioned Concrete Long-Span Slabs in Projects of Modern Building Construction* IOP Conf. Series: Mater. Sci. Eng. **245** 022065.
[3] Mieszczak M, Domagała L 2018 *Mater. Sci. Forum* **926** 140-5.
[4] The Concrete Society 2005 *Post-tensioned concrete floors. Design Handbook* Tech. Report No. 43, Cromwell Press, Wiltshire, UK